



## RESISTIVITY METHODS – MT

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### ABSTRACT

Magnetotellurics (MT) is a passive electrical resistivity method that probes the subsurface resistivity structure from a depth of a few tens of meters down to a depth of several tens or even hundreds of km. It is the only method that is capable of revealing the resistivity at such a great depth. It is a passive method, using the magnetic variations of the geomagnetic field as its source. Contrary to active resistivity methods like, the geoelectrical methods where an injected current into the earth is the source and the TEM method where the source is a current loop.

The basic knowledge of the MT method includes acquisition, processing and interpretation. There exists a variety of books and articles on the method which has developed greatly the last decades as the dynamical range in data recording has increased and computer capacities as well. A book that covers the practical aspects of applying the magnetotelluric technique should be recommended: Practical Magnetotellurics by Simpson and Bahr (2005). Making use of the internet is always a good idea. Take a look at the website, MTnet which is an internet forum for the free exchange of programs, data and ideas, especially for scientists in non-commercial studies. The address for the web page is: [www.mtnet.info](http://www.mtnet.info).

### 1. INTRODUCTION

The magnetotelluric (MT) method is a passive surface measurement of the earth's natural electrical (**E**) field and magnetic (**H**) field in orthogonal directions. It can be shown that the relationship between the horizontal orthogonal magnetic and electric fields depends on the subsurface resistivity structure. It is therefore used to determine the conductivity of the earth, ranging from a few tens of meters to several hundreds of kilometers. The fundamental theory was first developed by Cagniard (1950) and Tikhonov (1950, reprinted 1986). MT generally refers to recording of 10 kHz to 1000 s (0.001 Hz) or as low as 10.000 s (0.0001 Hz). AMT (audio MT) refers to „audio,, frequencies, generally recording > 100 Hz to 10 kHz. LMT-Long period MT generally refers to recording from 1.000 s to 10.000 s or much higher (to 100.000). The depth of penetration of MT soundings depends on the frequency, the lower the frequency the greater the depth of penetration and vice versa.

The earth's electromagnetic field contains a wide spectrum (Figure 1). The low frequencies are generated by ionospheric and magnetospheric currents caused by solar wind (plasma) interfering with the earth's magnetic field. Higher frequencies (< 1 Hz) are due to thunderstorms near the equator distributed as guided waves between the earth and the ionosphere. The time varying magnetic field

induces electric field and hence currents in the ground. By measuring variations in the magnetic and electric fields in the surface of the ground, information on the subsurface resistivity can be obtained.

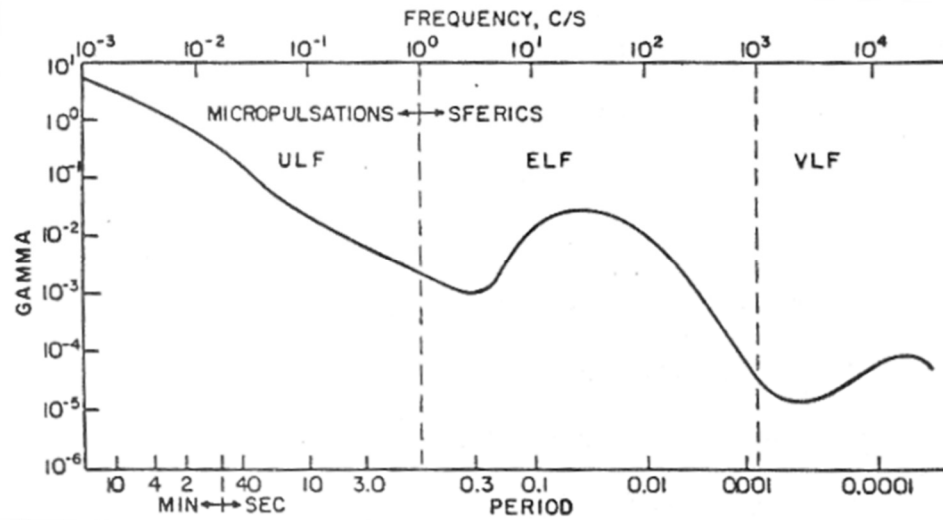


FIGURE 1: The natural magnetic field spectrum

The sunspot cycle has been measured since the mid 17th century. There is a regular cycle of little over 10 years. Presently there is a minimum of activity (see Figure 2), making MT soundings difficult in present years.

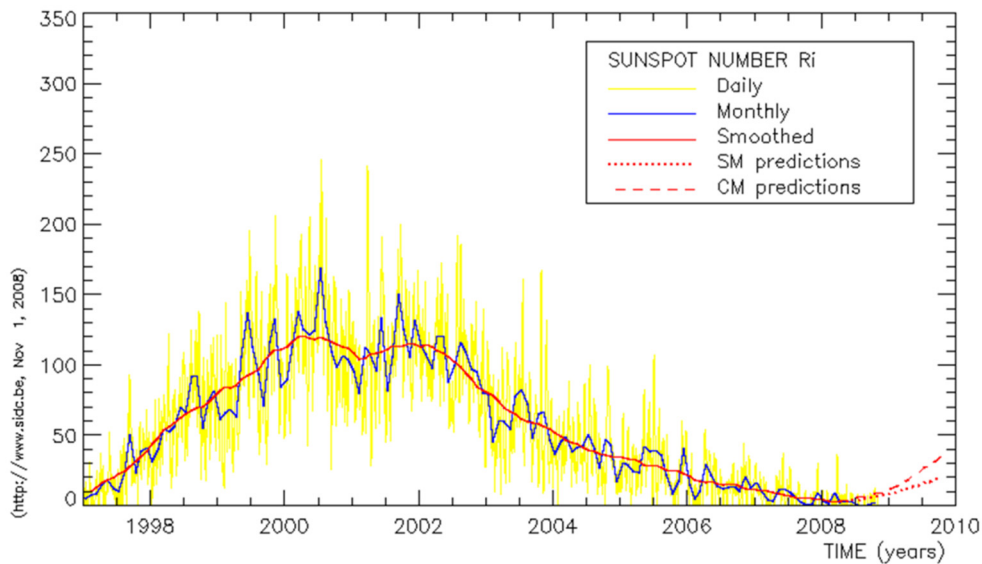


FIGURE 2: Current sunspot cycle

A typical setup for an MT sounding is shown in Figure 3. The horizontal orthogonal magnetic field,  $H_x$  (usually aligned in the magnetic north-south direction) and  $H_y$  (perpendicular to  $H_x$ ) and the vertical magnetic field  $H_z$  are measured by magnetic coils. The horizontal orthogonal electric field,  $E_x$  and  $E_y$  are measured by a pair of electrodes (the potential difference divided by the distance, 50-100 m). A GPS unit is used for synchronizing the data. The digital recording of the electromagnetic fields as a function of time is done through an acquisition unit and the time series saved on a memory card. The time series are Fourier transformed from the time domain to the frequency domain and processed. The tensors and the consequent apparent resistivity and phase are calculated as a function of the frequencies for later to be interpreted into a resistivity model of the subsurface (see Figure 5).

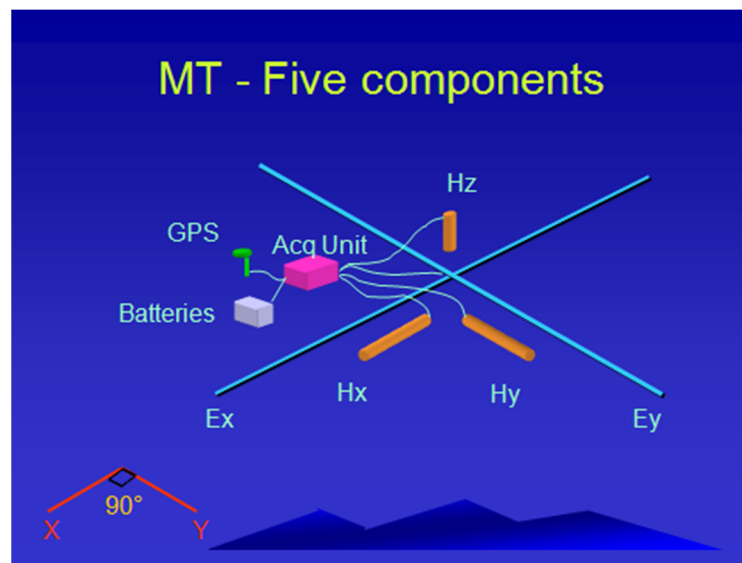


FIGURE 3: The setup of a magnetotelluric sounding

## 2. GENERAL THEORY OF ELECTROMAGNETISM

The four Maxwell's equations describe the electromagnetic field. They are Gauss's law for the magnetic field ( $\text{div}\mathbf{B}=0$ ) and Gauss's law for the electric field ( $\text{div}\mathbf{D}=q$ ), Faraday's law and Ampère's law with Maxwell's term:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \quad \text{Faraday's law}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \quad \text{Ampere's law}$$

$\mathbf{E}$ : Electrical field intensity (V/m)

$\mathbf{H}$ : Magnetic field intensity (A/m)

$\mathbf{J}$ : Electrical current density;  $\mathbf{J}=\sigma\mathbf{E}$

$\sigma$ : conductivity (Simens/m);  $\rho=1/\sigma$  ( $\Omega\text{m}$ )

$\varepsilon$ : electrical permittivity

$\mu$ : magnetic permeability

Assuming:

•Harmonic dependence of the oscillating electromagnetic fields;  $\mathbf{H}, \mathbf{E} \sim e^{i\omega t}$ ;

$\omega=2\pi\nu$  (angular frequency),  $i=\sqrt{-1}$

$\nu$  is frequency;  $\nu=1/T$ ;  $T$  is period;

gives :  $\frac{\partial \mathbf{H}}{\partial t} = i\omega \mathbf{H}$ ;  $\frac{\partial \mathbf{E}}{\partial t} = i\omega \mathbf{E}$

•Vertically incident plane wave

i.e.  $\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$

Using Faraday's law above, gives:

$$\begin{aligned}\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= -\mu \frac{\partial H_x}{\partial t} \Rightarrow -\frac{\partial E_y}{\partial z} = -\mu i \omega H_x \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= -\mu \frac{\partial H_y}{\partial t} \Rightarrow \frac{\partial E_x}{\partial z} = -\mu i \omega H_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -\mu \frac{\partial H_z}{\partial t} = 0\end{aligned}$$

Similarly applying Ampère's law above, gives:

$$\begin{aligned}\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= \sigma E_x + \varepsilon \frac{\partial E_x}{\partial t} \Rightarrow -\frac{\partial H_y}{\partial z} = (\sigma + i \omega \varepsilon) E_x \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= \sigma E_y + \varepsilon \frac{\partial E_y}{\partial t} \Rightarrow \frac{\partial H_x}{\partial z} = (\sigma + i \omega \varepsilon) E_y \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= \sigma E_z + \varepsilon \frac{\partial E_z}{\partial t} = 0\end{aligned}$$

Combining Faraday's and Ampère's law gives:

$$k^2 = i \omega \mu (\sigma + i \omega \varepsilon)$$

$$\sigma = 1/\rho; \text{ and } \rho \approx 1-10^4 \text{ } \Omega\text{m, or } \sigma \approx 1-10^{-4} \text{ S/m}$$

$$\omega = 2\pi/T \text{ and } T \approx 10^{-4} - 10^4 \text{ seconds}$$

$$\varepsilon = \varepsilon_0 \chi_e; \quad \varepsilon_0 = 8.85 \cdot 10^{-12}, \text{ and } \chi_e \approx 1-100$$

$$\Rightarrow (\omega \varepsilon)_{\max} = 2\pi \cdot 10^4 \cdot 8.85 \cdot 10^{-12} \cdot 100 \approx 5 \cdot 10^{-5}$$

$$\Rightarrow \sigma \gg \omega \varepsilon$$

$$\Rightarrow k^2 \approx i \omega \mu \sigma$$

For the quasi-stationary approximation:

$$\frac{\partial^2 E_y}{\partial^2 z} = i \omega \mu \frac{\partial H_x}{\partial z} = i \omega \mu (\sigma + i \omega \varepsilon) E_y$$

and:

$$\frac{\partial^2 E_x}{\partial^2 z} = -i \omega \mu \frac{\partial H_y}{\partial z} = i \omega \mu (\sigma + i \omega \varepsilon) E_x$$

Which can be written:

$$\frac{\partial^2 E_x}{\partial^2 z} = k^2 E_x; \quad k^2 = i \omega \mu (\sigma + i \omega \varepsilon)$$

$$\frac{\partial^2 E_y}{\partial^2 z} = k^2 E_y$$

The general solutions for a homogeneous earth ( $\sigma$  constant) can be written as:

$$\begin{aligned} E_{x,y} &= (A_{x,y}e^{kz} + B_{x,y}e^{-kz})e^{i\omega t} \\ H_x &= \frac{1}{i\omega\mu} \frac{\partial E_y}{\partial z} = \frac{k}{i\omega\mu} (A_y e^{kz} - B_y e^{-kz}) e^{i\omega t} \\ H_y &= \frac{-1}{i\omega\mu} \frac{\partial E_x}{\partial z} = \frac{-k}{i\omega\mu} (A_x e^{kz} - B_x e^{-kz}) e^{i\omega t} \end{aligned}$$

$A_{x,y}$  and  $B_{x,y}$  are constants to be determined

As  $z \rightarrow \infty \Rightarrow H \rightarrow 0$  and  $E \rightarrow 0 \Rightarrow A_{x,y} = 0$ ; and we get:

$$\begin{aligned} E_x &= B_x e^{-kz} e^{i\omega t} \quad ; \quad E_y = B_y e^{-kz} e^{i\omega t} \\ H_x &= \frac{-k}{i\omega\mu} B_y e^{-kz} e^{i\omega t} = \frac{-k}{i\omega\mu} E_y \\ H_y &= \frac{k}{i\omega\mu} B_x e^{-kz} e^{i\omega t} = \frac{-k}{i\omega\mu} E_x \end{aligned}$$

The impedance tensor elements  $Z_{ij}$  are defined as:

$$\begin{aligned} Z_{xy} &= \frac{E_x}{H_y} = \frac{i\omega\mu}{k} \approx \frac{i\omega\mu}{\sqrt{i\omega\sigma}} = \sqrt{\omega\mu\rho} \cdot e^{i\pi/4} \\ Z_{yx} &= \frac{E_y}{H_x} = \frac{-i\omega\mu}{k} = -Z_{xy} \end{aligned}$$

and we can calculate the resistivity of the half -space

$$\rho = \frac{1}{\omega\mu} |Z_{xy}|^2 = \frac{1}{\omega\mu} |Z_{yx}|^2$$

The resistivity may also be written as:

$$\rho = \frac{1}{\omega\mu} \left| \frac{E}{H} \right|^2 = \frac{T}{2\pi\mu} \left| \frac{E' \cdot 10^{-6} \cdot \mu}{B' \cdot 10^{-9}} \right|^2 = \frac{T\mu}{2\pi} \left| \frac{E'}{H'} \right|^2 \cdot 10^6 = 0.2T \left| \frac{E'}{H'} \right|^2$$

For a non homogeneous earth the apparent resistivity ( $\rho_a$ ) and phase ( $\Theta_a$ ) are defined as.

$$\rho_a = \frac{1}{\omega\mu} |Z_0|^2 \quad ; \quad \theta_a = \arg(Z_0) \neq 45^\circ$$

$Z_0$  is impedance at surface

The general definition of the impedance tensor is:

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix} = \vec{Z}\vec{H}$$

For 1D earth,  $Z_{xx} = Z_{yy} = 0$  and  $Z_{xy} = -Z_{yx}$ .

For 2D earth it is possible to rotate the tensor in the strike direction and  $Z_{xx} = Z_{yy} = 0$ .

For 3D earth it is not possible to rotate the tensor and get  $Z_{xx} = Z_{yy} = 0$ .

Figure 4 shows the apparent resistivity and phase as a function of the period for an MT station. The two resistivities  $\rho_{xy}$  and  $\rho_{yx}$  are not equal. The resistivity structure is therefore not 1D. The strike direction ( $Z_{strike}$ ) is around  $20^\circ$ , giving an indication of a 2D structure, since  $Z_{strike}$  is more or less the same value for all frequencies and the skew is low. The coherency is close to one, which means that there is a good correlation between the electric and magnetic fields.

FIGURE 4: Apparent resistivity and phase,  $Z_{strike}$ , Skew and coherency for an MT sounding (taken from Hersir et al., 2009)

Figure 5 shows the joint inversion of the MT sounding and a TEM sounding measured at the same location.

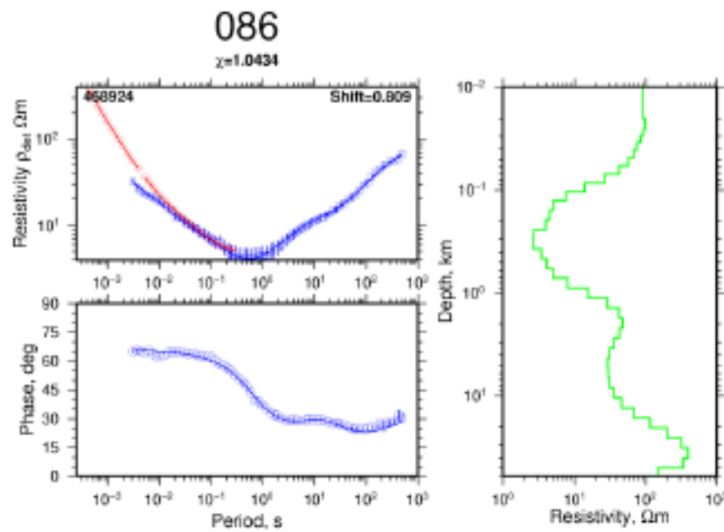


FIGURE 5: Joint inversion of TEM MT data. Red dots denote TEM data and the blue ones MT data. The resistivity model is shown in green to the left (taken from Hersir et al., 2009)

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