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Optimizing the design of hydropower stations

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Abstract

State of the art: Local optimization is common knowledge but leads to difficult calculations and time consuming search for an optimum. Global optimization is difficult in spreadsheets or matlab software but becomes easy in a software using Genetic Algorithms. An Objective Function is built by finding the net present value of the future income from energy sales and subtracting the financial and running costs. The basic equations are studied; different objectives, resource utilization policies and environmental considerations are discussed with respect to their influence on the result. In the final phase a Genetic Algorithm routine search out an optimum and this is demonstrated on a case study example.

Introduction

Local optimization of individual structures such as tunnel diameters is common knowledge in design of hydropower stations. The calculations are very difficult however. The search for an optimum is also very time consuming.

Global optimization is very difficult, even fully computerized in spreadsheets or matlab programs. The software application HYDRA overcomes this difficulty by using Genetic Algorithms.

The HYDRA software produced by Univ. of Icel. in cooperation with NPCI and Tech. Univ. of Vienna. In the reference list the 5 first references discuss this application. In this lecture is focusses on the special principles that are the working mechanism of this application and also shows a case study.

One principle used, is the mathematical maximization of an objective function is (Eliasson *et al* 1997):

$$\max f(x_1, x_2,\dots,x_n) \quad a_i \le x_i \le b_i \text{ for } i = 1 \text{ to } n \tag{1}$$

$$g_j(x_1,x_2,\dots,x_n) \le c_j \quad \text{for } \forall\ j \tag{2}$$

In this chapter the principle of optimal profit is introduced as our objective, so $f(x_1,x_2,\dots,x_n)$ is the profit, depending on the vector $(x_1,x_2,\dots,x_n)$, that stores all the necessary variables needed to compute the power production and investment costs. This leads to a method that in fact includes many of the conventional local optimization methods used so far, and can yield the same results.

By assuming an infinite energy demand and a fixed energy price, k_e, the present value of the revenue of energy sale becomes (Eliasson & Ludvigsson 1996):

$$NPV = T - G = k_e \cdot E \cdot \left(\frac{1-(1+r)^{-N}}{r}\right) - C \cdot v \cdot \left(\frac{1-(1+r)^{-N}}{r}\right) - C \tag{3}$$

Where T is total income, G gross expenses, r is the interest rate, N is the lifetime of the investment, C is the project investment, ν is the annual operation and maintenance cost, k_e is the unit price of energy, and E is the annual energy capacity of the hydrostation.

As all costs and revenues are included in the objective function, the optimization can be considered global. Inserting NPV for $f(x_1, x_2,\dots,x_n)$ in (2) gives:

$$dNPV = \sum_{i=1}^{n} \frac{\partial NPV}{\partial x_i} dx_i = 0 \Rightarrow \frac{\partial NPV}{\partial x_i} = 0 \text{ for all } i = 1 \text{ to } n \tag{4}$$

The optimization that Mosonyi (1991), and since then other authors, presents for tunnels, may be deduced from (4). This is local optimization. Often, variable costs of other project items than the conduit itself are not taken into account, which results in a larger tunnel diameter than the optimal one.

Example 1: Local optimization of x_1, the diameter of a headrace tunnel

$$\frac{\partial NPV}{\partial x_i}=0 \qquad NPV = k_e \cdot E \cdot \left(\frac{1-(1+r)^{-N}}{r}\right) - C \cdot v \cdot \left(\frac{1-(1+r)^{-N}}{r}\right) - C$$

Speculations on how long the economic lifetime N should we and what the interest rate r and the annual maintenance cost v should be is outside our topic so we put N very large (40 – 60 years), v < r < 10 % and get:

$$NPV = \frac{k_e}{r} \cdot E \cdot - C \cdot \frac{v \cdot + r}{r} \Rightarrow (k_e \cdot E \cdot - C \cdot (v \cdot + r))\frac{1}{r} = (k_e \cdot E \cdot - C \cdot a)\frac{1}{r}$$

The tunnel diameter x_1 only affect the energy losses in the E term and the construction costs of the tunnel itself in the C term. We differentiate partially with respect to x_1:

$$\frac{\partial NPV}{\partial x_i} = 0 \Rightarrow k_e \frac{dE}{dx_1} = a\frac{dC}{dx_1}$$

We take x_1 to be the diameter of the tunnel. Now E depends on $A = \pi\, x_1^2$, the tunnel area, as smaller tunnel diameter means greater flow resistance and less energy output. We also take $C = k_3 A\ L$ where L is the tunnel length, k_3 the tunnel construction cost per cubic meter so construction costs decrease with decreasing tunnel diameter. Somewhere there must be an optimum.

Choosing Chezy's formula to represent the flow resistance will result in the following formula

$$A = \left(\frac{T_k \cdot k_e}{k_3 \cdot a} \cdot \frac{2{,}5 \cdot e \cdot \gamma \cdot Q_v^{\,3}}{C_e^{\,2} \cdot \mu}\right)^{2/7}$$

Where C_e is Chezy's coefficient of flow resistance, e is efficiency of power station, γ is unit weight of water and μ is the ratio of hydraulic radius over square root of A, it is 0,25 – 0,28 for most tunnel cross-sections.

What have we got here? It is one of many forms of the formula for optimal size of a headrace tunnel cross section in a hydroelectric power plant. Now several questions arise. First: is the formula explicit and ready to use ? Answer is no, k_3 and C_e depend slightly on x_1 (tunnel diameter) so at least we have to use some iteration. Second: are there limitations to the validity? Answer is yes, we have used simplifying assumptions to get through a very complicated part of the calculations, which is the relationship between flow resistance and annual energy production, for details we must refer to Eliasson 1997. Third: is the formula generally valid accepting the limitations and possible iterations? Answer is no, there is a cost item not included in the formula, which is the size of powerhouse and mechanical equipment.

The conclusion of this example is that even using complicated methods, local optimization can only produce implicit formulas of limited validity. Counterexamples do exist, but they are few and uninteresting.

Different objectives

The result of example 1 brings us back to the global problem of eq. (3). We may ask the question if the principle of profit optimization is really global enough, can it possibly include important objectives such as environmental consideration and the reasonable demand for cheap electricity for public utilities? Can these considerations be included in a profit maximizing objective function?

Environmental considerations have two sides, first there is the resource utilization principle, second the principle of nature conservation. These two sides will be discussed in examples 2 and 3.

Example 2: The principle of long-term marginal costs

In the first case we don't want the utilization of a certain amount of resource to spoil the resource. This can happen in harnessing hydroelectric energy. We never utilize 100 % of a resource, there is always something left, and what is left is usually uneconomical to use. Diminishing energy resources of the world have as consequence rising prices, energy resources that are uneconomical to harness today, maybe economical tomorrow. But it is usually uneconomical to enlarge old power plants, so considerable hydroelectric energy may go lost in the future if we build only small power plants today.

The principle of long-term marginal cost has been applied in Norway and Iceland. In short it says that a resource shall be utilized until the marginal cost, J kr/kwh, matches the long-term price for other (fossil) energy.

In order to understand this design principle imagine that we plan a power station with annual energy output E. Then we plan a little bit bigger power station with annual energy output E + dE. Assuming that our plan is the most economical way to achieve the enlargement dE, the principle of long term marginal cost tells us that the power station is big enough not spoil the resource if:

$$dC/dE = J$$

Then the size of the power station is right. If our dC/dE < J we have to try a bigger station, if dC/dE > J we have to make it smaller. How can this be included in our objective function eq. 3? ?

Differentiating eq. 3 with respect to E and putting the result equal to zero as in eq. (4) results in:

$$\frac{d}{dE}(k_e \cdot E - C(1+\nu))\left(\frac{1-(1+r)^{-N}}{r}\right) = 0 \Rightarrow k_e = \frac{dC}{dE}(1+\nu) = J(1+\nu)$$

Which shows that for the optimum of the objective function, k_e is equal to the marginal cost of energy instead of the power sales price. By simply replacing the power sales price with the long-term marginal cost (augmented for operation and maintenance) we change the objective from profit maximization to resource utilization. The conclusion of example 2 is that selecting the k_e different from the power sales price the objective function is changed from profit maximization to another objective, e.g. marginal cost design.

Example 3: Nature conservation.

Second class, or the second site, of environmental considerations is that the development must not harm the environment, the values of nature have to be conserved.

Total conservation is simple; law (conservation act) protects the project site and the project suspended. Several sites are protected this way in almost every country in the world. The respective area is usually made a national monument.

Partial conservation can be made in a number of ways. The most common is restrictions on land use (such as borrow pits and fill areas), restrictions on storage volumes in reservoirs or minimum (or maximum) flows in rivers and many other things. Such restrictions either enters the cost function directly through their influence on unit prices or as restriction on the vector $(x_1,x_2,\ldots,x_n)$ in eq. (2) and through that they become a natural element in the optimization process. E.g. if we are supposed not to let the water level in a storage reservoir not exceed a certain elevation H, and suppose the storage volume V(H) in this reservoir is $x_2 = V$, then we have

$$x_2 < V_{max}(H)$$

As a natural restriction in the optimization.
Usually there are various environmental obligations involved in the permit the developer must obtain prior to construction of the power plant. This may involve various cost items that are not functions of the vector $(x_1,x_2,\ldots,x_n)$ but independent of it and can therefore not be included in the optimization in the way we just did with the reservoir elevation. This may be items to protect and research wildlife or fish habitats or create public access to scenic areas. These cost items do not affect the optimization as such as they drop out in the differentiation in eq. (4), but they affect the resulting profit and may turn it negative and thus render the project unfeasible for the developer. In the case of marginal costs, they can keep the average cost above the marginal cost and thus rule the project out. In such a way environmental obligations can serve the same purpose as total conservation.
In short, including environmental obligations that protect the nature in the optimization eqs (1) – (4) is usually not a problem.

Global optimisation

The global optimization problem cannot be solved analytically, the nonlinear constraints eq. 2 rule this possibility out completely. Therefore the program HYDRA has been developed to solve the global optimization problem. It does so using genetic algorithms, but it belongs to the class of methods called evolutionary methods (Goldberg 1989). One does not have to understand how genetic algorithms work, it is sufficient to know that the method seeks out the optimum by giving the vector $\{ x_i \}$ a definite values, calculating C and comparing the results. This sounds as both impractical and time consuming, but the genetic algorithm seeks out the optimum and finds it with astonishing speed (Eliasson et al 1997[1], 1997[2], 1998 and 1999).

Example 4: Global optimization of simple powerplant
A simple example, shown in figure 1 (Eliasson et al 1997[1]). Eqs. 1 – 4 are derived by direct mathematical analysis and solved. To do so it was necessary to build special approximation formula for the powerhouse and other construction elements shown in fig 1. The mathematical solution is compared to the findings of the HYDRA program in table 1, NPV, for different number of individuals P, generations G and mutation probability μ. It is necessary to explain the parameters P, G and μ shortly.
The computer stores $\{x_i\}$ vectors as P individual strings in the memory. Profit is calculated for all of them and the best performing (highest profit) individuals selected, these are the "parents". By special mixing of the elements of the best vectors a new set of P individuals is formed, this set is a new generation the "children". Now the process is repeated G times. To prevent the process to get stuck in a local maxima brand new children, unrelated to the parents are formed randomly, the mutation probability μ decides how often this happens. When the process stops after G generations the optimum should be found.

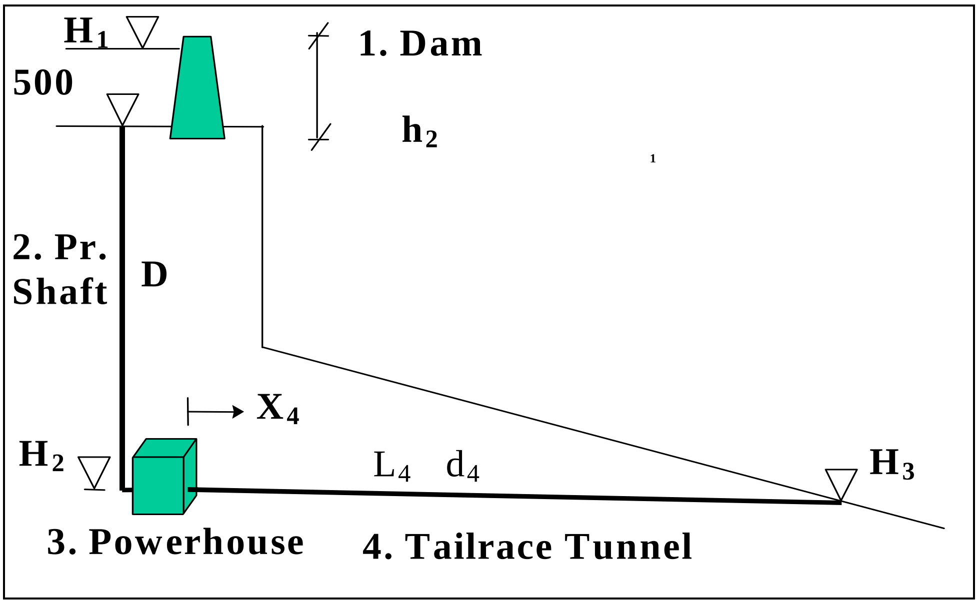

Fig. 1 Simple hydropower plant(Eliasson *et al* 1997[1]).

Table 1. Mathematical solution (bold) compared to optimisation results,

P		50	50	50	50	50	20	20
G		100	100	100	100	100	200	200
μ		0.001	0.005	0.01	0.025	0.05	0.025	0.05
D	**4,0**	4.0	4.0	4.0	3.9	4.0	4.0	3.9
H_1	**543,0**	543	543	543	543	543	543	543
H_2	**48,2**	44	49	49	48	42	50	44
H_3	**44,9**	39	46	46	45	37	46	39
NPV	**28594**	28580	28594	28594	28590	28569	28593	28576
dNPV	**-**	-14	0	0	-4	-25	-1	-18

The trick in this computation is to select P, G and μ so the optimum is truly found, without spending excessive computertime by selecting P, G and μ too high.
When the results of the optimization are compared with the mathematical solution, it is obvious that the runs where the P, G and μ parameters are optimally tuned reach results very close to the true optimum.

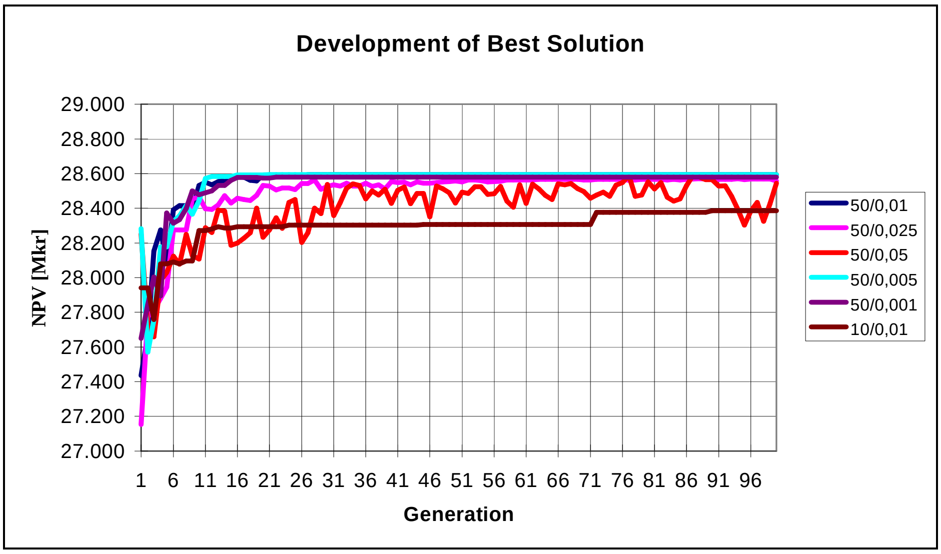

Figure 2. Development of solution for the different parameters in table 1
Eliasson *et al* 1997[1]

The result of the conventional local optimization method is also calculated and it gives an optimum diameter, D, of 4.5 m, which is a 0.5 m difference in the diameter between methods. The conclusion of example 3 is that genetic algorithm is a suitable tool for finding the optimal plant arrangement.

The HYDRA software is a shell that contains program objects that calculate the NPV of all construction elements (Eliasson *et al* 1997[2]). Points that have geographical coordinates connect them and these can be included in the optimization if necessary. Thus tunnel lengths and position of powerhouses can be found, see e.g. the tailrace tunnel in fig. 1 example 3. Here L_4 is optimized.

Experience shows that running times are in the vicinity of 2-4 minutes for very complicated hydropower plants, depending on the size of population and number of generations. G = 2P seems to be a suitable rule and in most cases P = 30 is enough. The suitable m is highly dependent upon P see fig. 1 example 3.

HYDRA has performed very well on very complicated project planning tasks (Eliasson *et al* 1998, Eliasson *et al* 1999[1] and Eliasson *et al* 1999[2]). Fig 3 shows the object diagram in Eliasson *et al* 1999. It is a good example of the complexity that can be handled by the program.

In the beginning all the objects in HYDRA used approximation formulas to calculate the NPV of their respective construction elements. Today cost estimates based on quantities and locally adjusted unit prices for concrete, dam fills, tunnel-driving etc. are used. An exception from this is the powerhouse but here the old formula is still in use. The guidelines for a more advanced powerhouse object have been given in chapter 5 in Eliasson *et al* 1999[1], and this important contribution is by professors Matthias, H. – B. and G., Doujak of Technical University of Vienna. G., Doujak, further elaborates the subject in chapter 4 in Eliasson et al 19992, this time.

To find the profit E has to be calculated. This has to be the expected power output of the station. This calculation has to be performed for each individual in each generation. This is done by using the load factors derived in (Eliasson 2000). The load factors have to be putted into HYDRA in the beginning of each run. The method to find them is very complicated and will not be repeated here.

Global optimization results, theoretical remarks

Some information is included in the optimization in an implicit manner and has to be extracted by means of theoretical considerations. Lets take a few examples.

In theory, we search for maximum profit in the optimization. We have not considered the average cost of power per kwh the utility can offer the customers. We have only considered a fixed market price but a utility may want to offer cheaper electricity to its customers. What can be done?

Example 5: Average power cost and opportunity cost

For the utility the average power cost is

$$k_0 = \frac{G_a}{E} = \frac{G_a}{T_a / k_e} \cong k_e \frac{G}{T} \Rightarrow k_0 = k_e \frac{G}{G + \kappa H}$$

Here G_a and T_a are annual values in contrast to the NPV’s G and T. The factor κ is to take care of that the ratio of annual values is not exactly the same as the ratio of NPV´s. A common value of κ is between 0.9 and 1.1

If we make a series of optimizations for lower and lower sales price k, the profit H will tend to zero and in that limit we get:

k_o = ke

Or the average power cost is equal to the unit energy sales price. This is called the break-even price or the opportunity cost price. It is the lowest unit price one can offer for a commodity without loosing money.

Case studies

Skagafjordur Iceland

The scheme is described in (Eliasson *et al* 1998). It consists of a storage reservoir (Austurbugur Storage = AS) and one or two powerhouses downstream. The reservoir is considered to release a fixed discharge to the powerhouse turbines. The discharge is calculated from the project capacity, a load factor of 0.816 is used. Economical benefits are calculated from power sales and surplus capacity that can act as a spinning reserve according to (prices in US dollars):

Primary power	0.03	$/kWh
Spinning reserve	7.14	$/kW/a

The investment cost includes various owner's costs as % of construction costs:

Camp and Mobilisation	6,2 %
Contingencies	20 %
Supervision and Engineering	12,66 %
Preliminary Investigation Cost	2,33 %
Other Owner's Cost	4,18 %
Interest During Construction	17,83 %

Table 2 Various owner's costs

Annual operation and maintenance is 0,8% of the project investment. The interest rate in NPV calculation is 6% and project economic lifetime 40 years.

Eventual use of thermal reserves has the following tariff:
Thermal reserve power 0.06 $ /kWh
Standby thermal reserve 17.14 $/kW/a
The main results of the optimisation compared to Fljotsdalsvirkjun are as follows:

Project	I.c.[1]	Power	Inv.c.[2]	B.e.p.[3]	C.cost[4]	Po.cost	A.stor. (AS)	
	Mw	Gwh/a	M$	$/kwh	M$/Mw	$/kwh/a	m.a.s.l	M m^3
Merkigilsvirkjun	176	1.259	355	0.021	2.01	0.286	708	282
SV / Villinganes	123	879	283	0.024	2.30	0.314	709	179
Giljamúli	103	735	237	0.024	2.30	0.329	704,5	215
Fossárvirkjun	92	655	245	0.028	2.65	0.371	705	223
Fljotsdalsvirkjun	*176*	*1.259*	*328*	*0.019*	*1.86*	*0.257*		

[1]Installed capacity [2]Investment costs [3]Break even price [4]Capacity cost

Table 3 Comparison of project economies

Notice in table 3, that each arrangement gives a different maximum elevation of the Austurbugur storage (AS), which clearly shows that the optimal size of the storage is very much dependent on other parts of the scheme.
Following answers to the principal questions concerning the project were given:
<u>How much power can be economically exploited in the region?</u>
Merkigilsvirkjun gives 176 MW installed capacity with anticipated production of 1259 Gwh/a. This is about 60 % of the technically harnessable potential in the area.
<u>The locations and the dimensions of main construction items.</u>
Main features are a headrace tunnel 46 km long and 4.9 m wide, and a pressure shaft 366 m deep and 2.7 m wide.
<u>What is the construction cost of the respective power stations.</u>
The construction costs are 2.0 – 2.3 M$/Mw, a very competitive price.
<u>How does the economy of individual projects compare to Fljotsdalsvirkjun</u>. Skagafjoerdur hydro is 10 – 20% more expensive than the Fljotsdalur project.
<u>What further field investigations will be necessary</u>
This is as follows:

- More information is needed on the effect of rhyolite on tunnelling conditions.
- Unmapped spots on Nyjabaejarfjall and west of the junction of the Austari Joekulsa river and the Vestari Joekulsa river need to be mapped.
- Streams flowing from the Nyjabaejarfjall area to Austari Joekulsa have to be gauged and their discharge estimated.
- Environmental investigations need to be started as soon as possible.

The last point is because this hydropower potential is indeed very attractive so eventual adverse effects of the exploitation on the environment have to be uncovered as quickly as possible.
It is remarkable that the most economical project is the Merkigilsvirkjun project. From this we can draw the inference that field investigations in the Merkigilsvirkjun area, that is in the eastern part of the Skagafjoerdur catchment, have to be given high priority.

Fljotsdalur

This example is taken from (Eliasson *et al* 1997 and 1999[1] and 1999[2]).

Table 4. Premliminary optimisation 1997

Description		PPR_1	PPR_2	O_{1150}	O_∞
Reservoir level	m.a.s.l.	664.5	668.5	665.1	667.6
Headrace tunnel dia.	m	5.0	5.0	4.3	4.8
Pressure shaft dia.	m	2.9	2.9	2.6	2.7
Power	MW	213	239	211	233
Energy	GWh/a	1159	1300	1150	1278
Investment	BIKR	21.16	22.91	19.92	21.96
Profit	BIKR	10.90	13.44	12.28	13.86
ΔProfit/ΔInvestment	% / %	0/0	+23/+8	+13/-6	+27/+4

Optimised dimensions are bold faced. 60 BIKR ≅ 1 billion $ in 1997

The O_{1150} optimisation seeks a slightly higher dam (increased discharge to the plant) to compensate for increased power losses in narrower conduits.

The O_∞ optimisation results in a significantly higher dam compared to the PPR_1. The explanation is that in the project planning report, the size of the power plant and the size of reservoir is selected on basis of a power market scenario at the expected construction time of the plant, but the optimisation assumes infinite demand. The solution is however not far from the PPR_2 arrangement.

The global optimisation O_{1150}, leads to a 0,7 m narrower headrace tunnel compared to the PPR. Local optimisation, considering only variable cost of the headrace, leads to the same result as in the PPR (5 m). The power capacity reduction due to increased headlosses, is compensated with a slightly larger reservoir (increased discharge).

The O_∞ optimisation results in a slightly smaller headrace diameter compared to the PPR. It is quite natural when compared to O_{1150}, that this optimisation seeks a larger tunnel, because there are no market restrictions.

The same logic can be used to explain the difference in the pressure shaft diameter. There is, however, a problem with the maximum velocity in the shaft. In both optimisations the diameter breaks the design criteria that the maximum velocity should be below 8 m/s. For both O_{1150} and O_∞, the minimum diameter that satisfies this constraint should be selected by the user, in both cases close to d = 2,8 m, depending on design discharge. This has a minor economical significance in this case, but is however a good example of how dependent constraints $g(x, y) \geq 0$ have to be considered in the future development. The way to handle this is to develop and add a penalty function, $\Phi(x)$, to the construction cost of the pressure tunnel type object (and other objects where necessary), that 'penalizes' the tunnel if it's water velocity exceeds the allowed value but is otherwise zero. This prevents the Genetic Algorithm from breaking this constraint.

The O_∞ optimisation results in a larger energy output than in the PPR_1. This is natural, as this optimisation assumes plant stage, which means no market restrictions and no extra benefit for the system. The extra benefit is that interactions between Fljótsdalur Power Plant and the existing power system produces substantial extra energy (estimated 250 GWh/a firm energy in the PPR) through better utilisation of the water resources.

The project investment is 6% lower in optimisation O_{1150} compared to the PPR_1, resulting in a 13% higher profit, which is a significant improvement. The optimisation O_∞ on the other hand leads to a 4% higher investment and a 27% higher profit. When it is kept in mind that the

PPR_1 plans a future raising of the dam to reservoir level 668,5 m.a.s.l (Fljótsdalur Engineering Joint Venture 1991), the result of O_∞ is very close to the PPR_2 version.
In order to ensure the best possible result in the global optimisation the cost estimation of the whole scheme is completely revised. The VOS construction cost functions are removed and replaced with new cost functions, specially prepared by the engineering consultants (Helgason, pers. comm.).
Now similar runs as for the Plant Stage are performed. The results are presented in table 5.

Table 6. Tabulation of significant data and net profit of the investment
(optimised dimensions are bold faced). 60 BIKR ≅ 1 billion $

Description		PPR_1	PPR_2	O_{1150}	O_∞
Reservoir level	m.a.s.l.	664.5	668.5	665.1	669.6
Headrace tunnel dia.	m	5.0	5.0	4.3	5.3
Pressure shaft dia.	m	2.9	2.9	2.6	2.8
Power	MW	212	239	210	242
Energy	GWh/a	1159	1300	1149	1325
Investment	BIKR	22.78	24.40	22.18	24.91
Profit	BIKR	9.36	11.78	9.72	11.97
ΔProfit/ΔInvestment	% / %	0/0	+26/+7	+4/-3	+28/+9

The O_{1150} optimisation leads to a similar arrangements as the plant stage optimisation. The O_∞ however shows significant changes. This is because the new cost formulas do not represent the true variation of the costs except in a narrow region around the PPR_1 values. Therefore the results of the O_∞ optimisation are hardly applicable. However a comparison of the columns O_∞ in Tables 4 and 5, shows how important it is that the cost formulas in the optimisation are accurate. It may therefore be concluded that it is worth the effort for the consultants, to take the time and trouble to have the cost formulas in Hydra improved with formulas specially designed by themselves, in order to improve the accuracy of optimisations performed.
The economical result is of course dominated by the overall increase in the construction cost, compared to the plant stage, which leads to a considerable decrease in the profit, probably meaning considerable decrease in the profit margin of venture capital.

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